# Horizontal Circular Curves Review

Blue Mountain Chapter PLSO
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presented by

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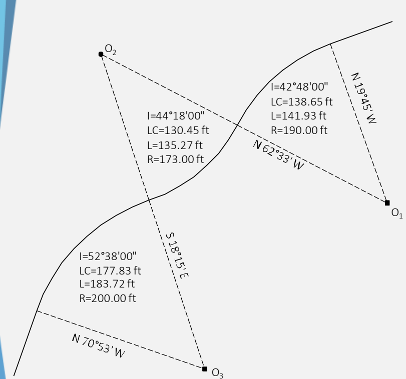


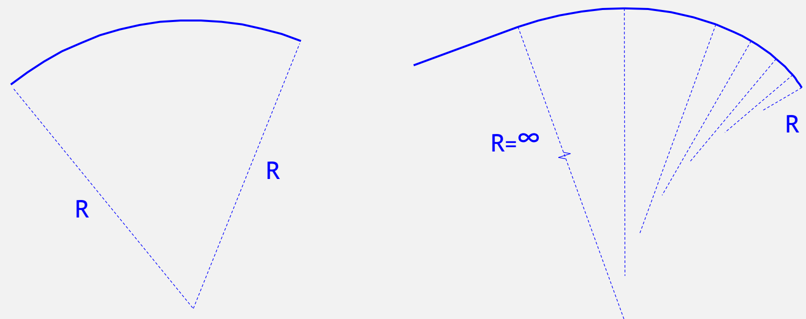

## Horizontal Curves

A. Introduction
B. Nomenclature
C. Equations
D. Tangency Conditions
E. Alignment Curves
F. Curves and Traverses
G. Problems

# A. Introduction

Geometric horizontal curves are either sections of circular arcs or spirals.

- The primary difference is the curve's radius
  - Circular arc: single constant radius throughout
  - Spiral: radius varies along the curve's length
- We'll concentrate on circular horizontal curves




Horizontal Curves

# A. Introduction

FS Exam Reference Handbook

**Horizontal Circular Curves**

$D$ = degree of curve, arc definition

$D_c$ = degree of curve, chord definition

$L$ = length of curve from P.C. to P.T.

$c$ = length of sub-chord

$\ell$ = length of arc for sub-chord

$d$ = central angle for sub-chord

$I$ or $\Delta$ = angle of interior or delta

$D = \frac{5,729.58}{R}$

Radius by chord definition, $R = \frac{50}{\sin\left(\frac{D}{2}\right)}$

$T = R\tan\left(\frac{I}{2}\right)$

$L = RI\frac{\pi}{180} = \frac{I}{D}(100)$

$LC = 2R\sin\left(\frac{I}{2}\right)$

$c = 2R\sin\left(\frac{d}{2}\right)$

$d = \ell D/100$

$M = R\left[1 - \cos\left(\frac{I}{2}\right)\right]$

$E = R\left[\frac{1}{\cos\left(\frac{I}{2}\right)} - 1\right]$

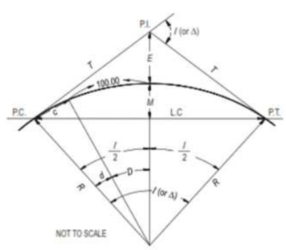
Area of sector $= \frac{RL}{2} = \frac{\pi R^2 I}{360}$

Area of segment $= \frac{\pi R^2 I}{360} - \frac{R^2 \sin I}{2}$

Area between curve and tangents $= R(T - L/2)$

$R = \frac{AC}{2\sin(a+b)}$

Equation of a circle, $X^2 + Y^2 = R^2$




PS Exam Reference Handbook

**HORIZONTAL CIRCULAR CURVES**

D – Degree of curve, arc definition

$D_c$ – Degree of curve, chord definition

L – Length of curve from P.C. to P.T.

c – Length of sub-chord

$\ell$ – Length of arc for sub-chord

d – Central angle for sub-chord

I or Δ – Angle of interior or delta

$D = \frac{5,729.58}{R}$

Radius by chord definition, $R = \frac{50}{\sin 1/2\,D}$

$T = R\tan(I/2)$

$L = RI\frac{\pi}{180} = \frac{I}{D}(100)$

$LC = 2R\ \sin(I/2)$

$c = 2R\sin(d/2)$

$d = \ell D/100$

$M = R[1 - \cos(I/2)]$

$E = R\left[\frac{1}{\cos(I/2)} - 1\right]$

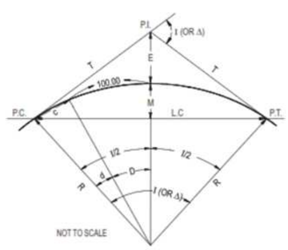
Area of sector $= \frac{RL}{2} = \frac{\pi R^2 I}{360}$

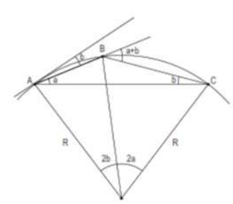
Area of segment $= \frac{\pi R^2 I}{360} - \frac{R^2 \sin I}{2}$

Area between curve and tangents $= R(T - L/2)$

$R = \frac{AC}{2\sin(a+b)}$

Equation of a circle, $X^2 + Y^2 = R^2$

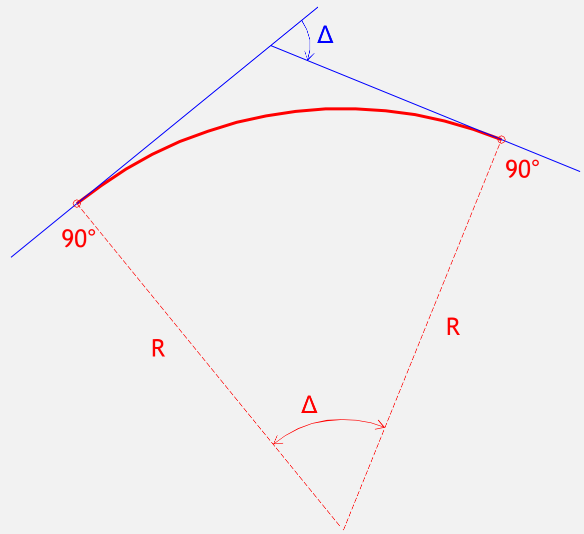



Horizontal Curves

## B. Nomenclature

- Δ – central angle of the curve
  - aka I-angle
  - Also, deflection angle of the tangent lines
- R – radius



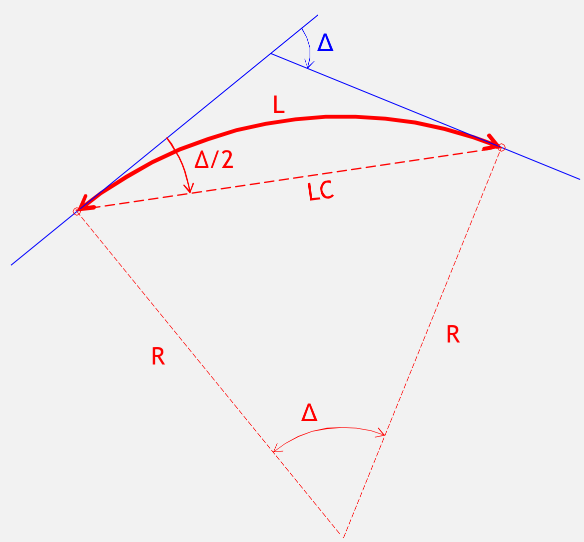

Horizontal Curves

## B. Nomenclature

- Δ – central angle of the curve
- R – radius

- L – curve Length
- LC – Long Chord

- Δ/2 - angle between tangent and long chord



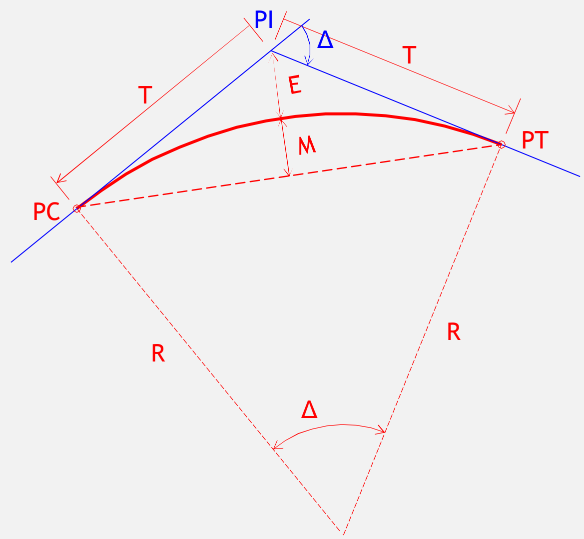

Horizontal Curves

## B. Nomenclature

Δ - deflection angle at PI
R - arc radius

L - curve Length
LC - Long Chord

T - Tangent Distance
E - External Distance
M - Middle Ordinate



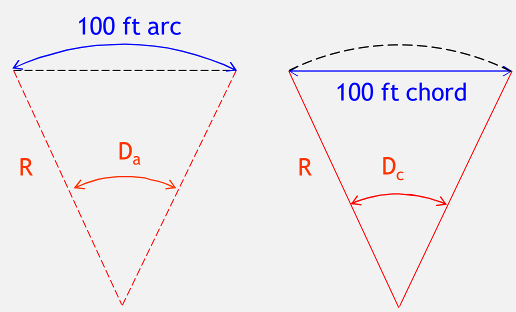

Horizontal Curves

## B. Nomenclature

Degree of Curvature - Indicator of curve sharpness

Two different kinds

- Arc Definition, $D_a$
  - Angle at center of 100.00 ft **arc**
    - Most common
    - Used for roads
- Chord Definition, $D_c$
  - Angle at center of 100.00 ft **chord**
    - Used for railroads




$$R = \frac{5729.58}{D_a}$$

$$R = \frac{50}{\sin\left(D_c/2\right)}$$

Horizontal Curves

## C. Equations

1. Components

$$L = 100\left(\frac{\Delta}{D}\right) = \frac{R\pi\Delta}{180}$$

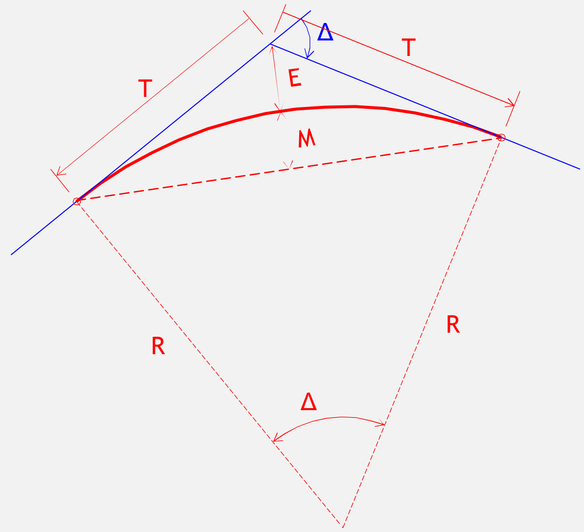
$$T = R \times \tan\left(\frac{\Delta}{2}\right)$$

$$LC = 2R \times \sin\left(\frac{\Delta}{2}\right)$$

$$E = R\left[\frac{1}{\cos\left(\frac{\Delta}{2}\right)} - 1\right]$$

$$M = R\left[1 - \cos\left(\frac{\Delta}{2}\right)\right] \qquad M \neq E$$

Two geometric elements must be defined to fix a curve




Horizontal Curves

## C. Equations

2. Areas

Fillet

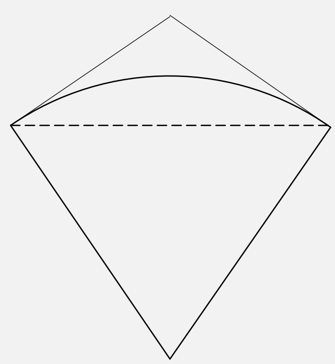
$$A = R^2\left[\tan\left(\frac{\Delta}{2}\right) - \frac{\Delta\pi}{360°}\right]$$

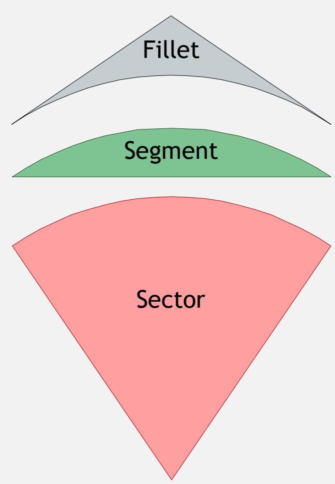
Segment

$$A = R^2\left[\frac{\Delta\pi}{360°} - \frac{Sin(\Delta)}{2}\right]$$

Sector

$$A = \frac{\Delta\pi R^2}{360°}$$

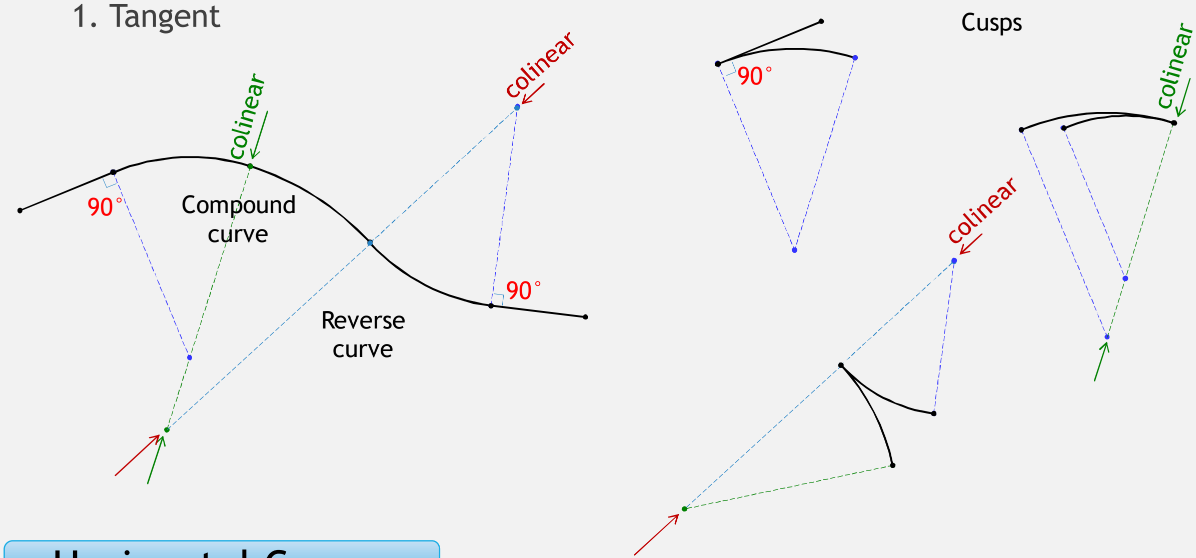





Horizontal Curves

## D. Tangency Conditions

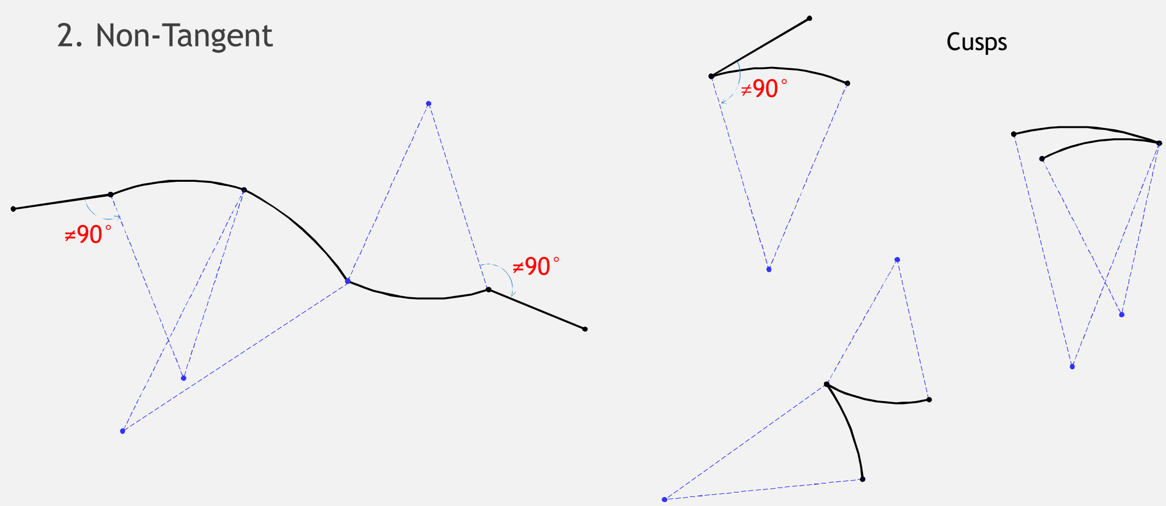
1. Tangent




Horizontal Curves

## D. Tangency Conditions

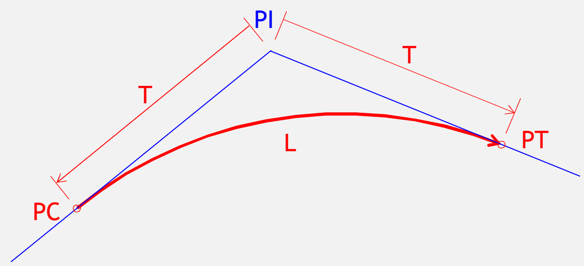
2. Non-Tangent




Horizontal Curves

## E. Alignment Curves

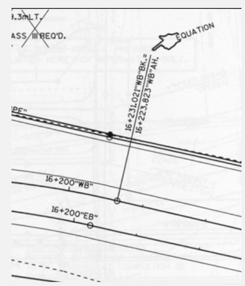
1. Tangent curves
   - Smooth line-curve-line transitions




2. Endpoint Stationing
   - PI: point of intersection
   - PC: point of curvature
     - $Sta_{PC} = Sta_{PI} - T$
   - PT: point of tangent
     - $Sta_{PT} = Sta_{PC} + L$
     - $Sta_{PT} = Sta_{PI} + T$

Station equation:
PT Back = PT Ahead

From traditional alignment staking




Horizontal Curves

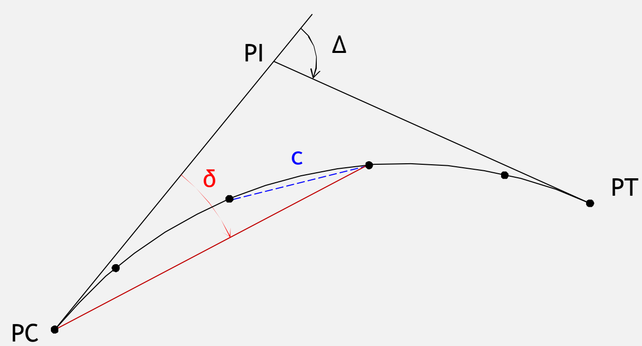
## E. Alignment Curves

3. Deflection Angle Method

Traditional method to compute and stake a curve

Deflection angle, δ: angle at PC from tangent to curve point.

Chord, c: straight distance between adjacent curve points




Horizontal Curves

## E. Alignment Curves

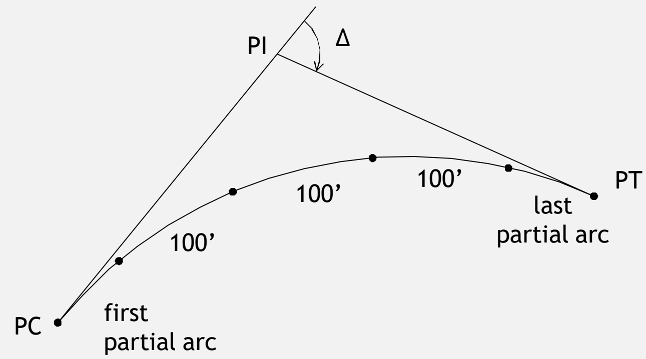
3. Deflection Angle Method

Compute curve at full (or half) stations.

PC & PT rarely at +00 station

So have partial arcs (< 100 ft) at begin and end of curve.

In between are 100 ft arcs




Horizontal Curves

## E. Alignment Curves

3. Deflection Angle Method

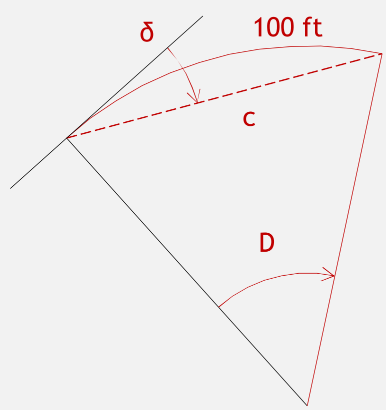
Deflection angle for an arc is half its central angle

Full station arc, l = 100 ft

$\delta = D/2$

Chord length:

$c = 2R \times \sin(D/2) = 2R \times \sin(\delta)$




Horizontal Curves

## E. Alignment Curves

3. Deflection Angle Method

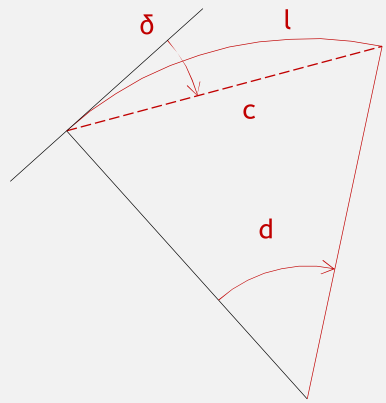
For partial arcs, l < 100 ft

$d = l(D/100)$

$\delta = d/2$

Chord length:

$c = 2R \times \sin(d/2) = 2R \times \sin(\delta)$




Horizontal Curves

## E. Alignment Curves

3. Deflection Angle Method

Successive deflection angles

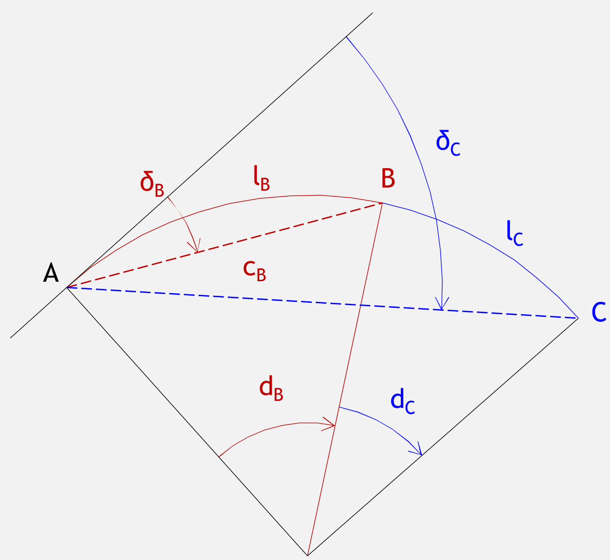
$\delta_B = d_B/2$

$\delta_C = (d_B+d_C)/2 = d_B/2 + d_C/2$

$= \delta_B + (d_C/2)$

$(d_C/2)$ is incremental deflection

Each defl angl increases by the inc defl




Horizontal Curves

## E. Alignment Curves

3 Deflection Angle Method

Example

Given

PI Sta = 27+10.00

Δ = 78°18'00" R

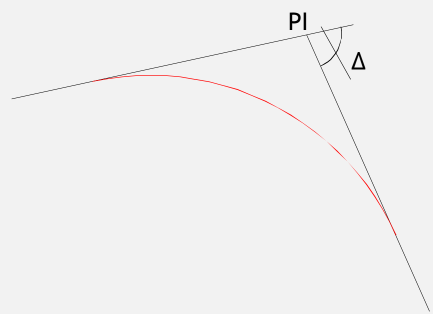
D = 18°00'00"

Compute

a. curve components

b. end point stations

c. deflection angles method to full stations



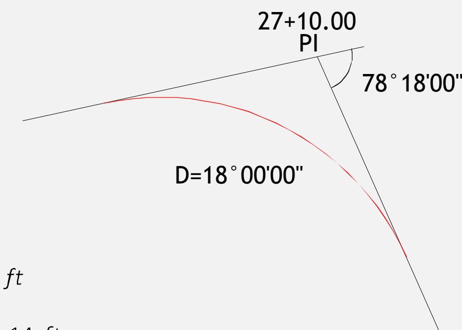

Horizontal Curves

## E. Alignment Curves

3 Deflection Angle Method

Example

a. Curve components




$$R = \frac{5729.58}{D} = \frac{5729.58}{18°00'00''} = 318.31\ ft$$

$$L = 100\left(\Delta/D\right) = 100\left(78°18'00''/18°00'00''\right) = 435.00\ ft$$

$$T = R \times \tan\left(\Delta/2\right) = 318.31 \times \tan\left(78°18'00''/2\right) = 259.14\ ft$$

$$LC = 2R \times \sin\left(\Delta/2\right) = 2 \times 318.31 \times \sin\left(78°18'00''/2\right) = 401.93\ ft$$

$$E = R\left[\frac{1}{\cos\left(\Delta/2\right)} - 1\right] = 318.31\left[\frac{1}{\cos\left(78°18'00''/2\right)} - 1\right] = 92.15\ ft$$

$$M = R\left[1 - \cos\left(\Delta/2\right)\right] = 318.31\left[1 - \cos\left(78°18'00''/2\right)\right] = 71.46\ ft$$

Horizontal Curves

## E. Alignment Curves

3 Deflection Angle Method

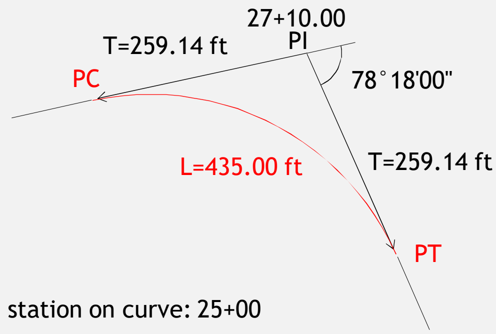
Example

b. Endpoint Stations

$$Sta_{PC} = Sta_{PI} - T$$
$$= (27+10.00) - 259.14$$
$$= 24+50.86$$
$$Sta_{PT} = Sta_{PC} + L$$
$$= (24+50.86) + 435.00$$
$$= 28+85.86 \quad Back$$
$$Sta_{PT} = Sta_{PI} + T$$
$$= (27+10.00) + 259.14$$
$$= 29+69.14 \quad Ahead$$

First full station on curve: 25+00

Last full station on curve: 28+00




Horizontal Curves

## E. Alignment Curves

3 Deflection Angle Method

Example

c. deflection angles to full stations

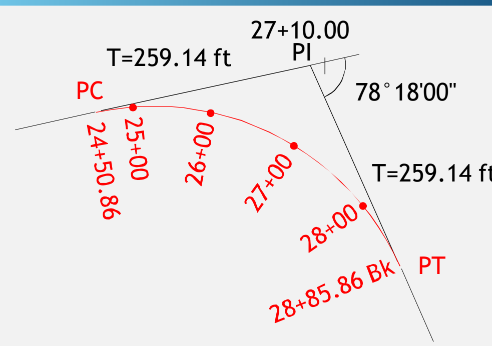
First partial arc

(25+00)-(24+50.86) = 49.14

Last partial arc

(28+85.86)-(28+00) = 85.86

100 ft arcs in between



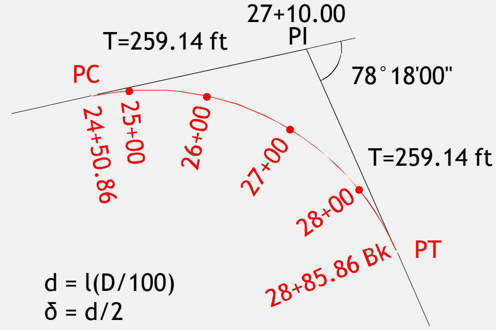

Horizontal Curves

# E. Alignment Curves

3 Deflection Angle Method

Example

c. deflection angles to full stations




d = l(D/100)
δ = d/2
c = 2R x sin(δ)

First partial arc: 49.14

$d_f$ = 49.14 x (18°00’00”/100) = 8°50’43”

$\delta_f$ = 8°40’46”/2 = 4°25’21”

$c_f$ = 2(318.31) x sin(4°25’21”) = 49.09

Last partial arc: 85.86

$d_l$= 85.86 x (18°00’00”/100) = 15°27’17”

$\delta_l$ = 15°27’14”/2 = 7°43’39”

$c_l$ = 2(318.31) x sin(7°43’39”) = 85.60

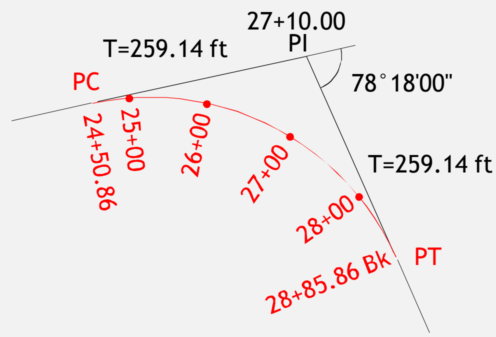
Horizontal Curves

# E. Alignment Curves

3 Deflection Angle Method

Example

c. deflection angles to full stations




Full arc: 100.00

D = 18°00’00”

δ = 18°00’00”/2 = 9°00’00”

$c_f$ = 2(318.31) x sin(9°00’00”) = 99.59

Horizontal Curves

# E. Alignment Curves

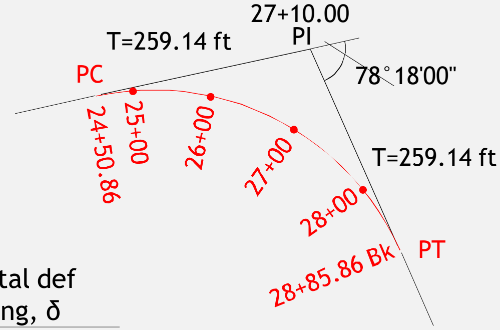
3 Deflection Angle Method

Example

c. deflection angles to full stations

Set up curve table

$\delta_i = \delta_{i-1}$ + Inc defl $ang_i$




| Sta | chord | Inc defl ang | Total def ang, δ |
|---|---|---|---|
| PT Bk 28+85.86 | 85.60 | 7°43’39” | |
| 28+00 | 99.59 | 9°00’00” | |
| 27+00 | 99.59 | 9°00’00” | |
| 26+00 | 99.59 | 9°00’00” | |
| 25+00 | 49.09 | 4°25’21” | |
| PC 24+50.86 | **0.00** | 0°00’00” | 0°00’00” |

Horizontal Curves

# E. Alignment Curves

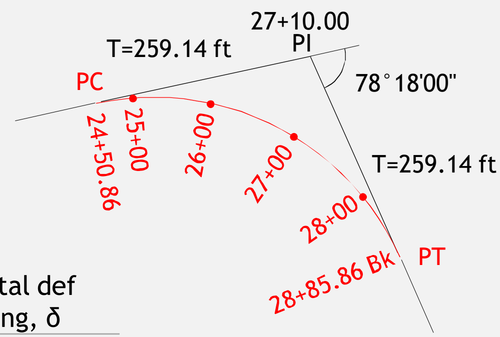
3 Deflection Angle Method

Example

c. deflection angles to full stations

Set up curve table

$\delta_i = \delta_{i-1}$ + Inc defl $ang_i$




| Sta | chord | Inc defl ang | Total def ang, δ |
|---|---|---|---|
| PT Bk 28+85.86 | 85.60 | 7°43’39” | |
| 28+00 | 99.59 | 9°00’00” | |
| 27+00 | 99.59 | 9°00’00” | |
| 26+00 | 99.59 | 9°00’00” | |
| 25+00 | 49.09 | 4°25’21” = | 4°25’21” |
| PC 24+50.86 | **0.00** | 0°00’00” + | 0°00’00” |

Horizontal Curves

# E. Alignment Curves

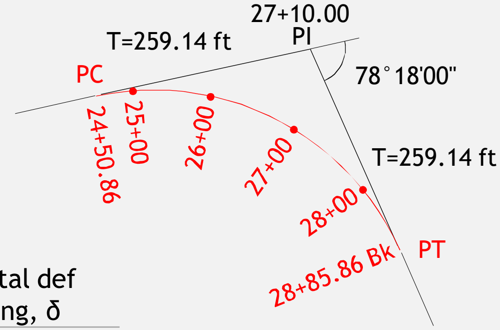
3 Deflection Angle Method

Example

c. deflection angles to full stations

Set up curve table

$\delta_i = \delta_{i-1}$ + Inc defl $ang_i$

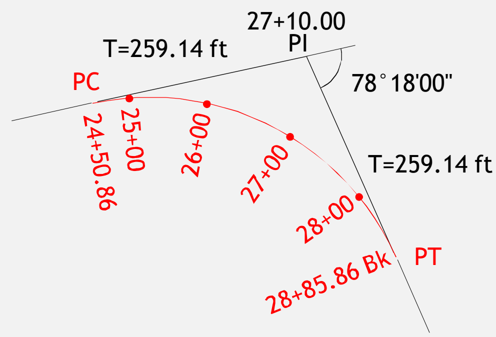



| Sta | chord | Inc defl ang | Total def ang, δ |
|---|---|---|---|
| PT Bk 28+85.86 | 85.60 | 7°43’39” | 39°09’00” =Δ/2 |
| 28+00 | 99.59 | 9°00’00” | 31°25’21” |
| 27+00 | 99.59 | 9°00’00” | 22°25’21” |
| 26+00 | 99.59 | 9°00’00” | 13°25’21” |
| 25+00 | 49.09 | 4°25’21” | 4°25’21” |
| PC 24+50.86 | **0.00** | 0°00’00” | 0°00’00” |

Horizontal Curves

# E. Alignment Curves

4. Modified Deflection Angle Method




Traditional method drawbacks:

(a) Inefficient - Requires 3 people

One sighting from PC

Two measuring chords

(b) Can’t skip a station if its sight is obstructed.

If can’t set one point, can’t set any after without recalculating or “moving up on the curve”

Horizontal Curves

# E. Alignment Curves

4. Modified Deflection Angle Method

Radial stake out from PC.

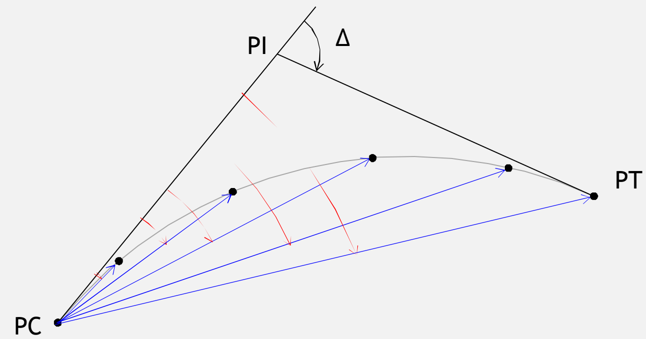
Chords measured radially from PC, not point-to-point on curve.

Advantages

(a) More efficient; fewer people needed.

(b) can skip curve points and still take subsequent points

(c) easier to calculate




Horizontal Curves

# E. Alignment Curves

4. Modified Deflection Angle Method

Curve has a constant deflection rate.

Defl angle for 100 ft arc is D/2

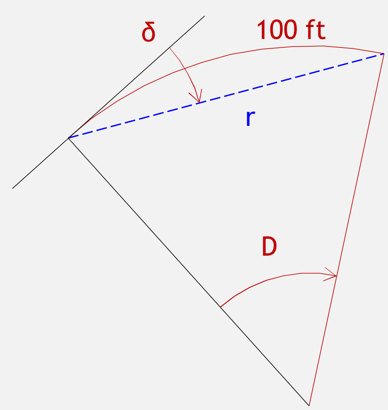
Defl rate = (D/2)/100 ft = D/200 ft

To any curve point:

$l_i = Sta_i - Sta_{PC}$

$\delta_i = l_i \times$ defl rate

$r_i = 2R \sin(\delta_i)$




Horizontal Curves

# E. Alignment Curves

## 4. Modified Deflection Angle Method

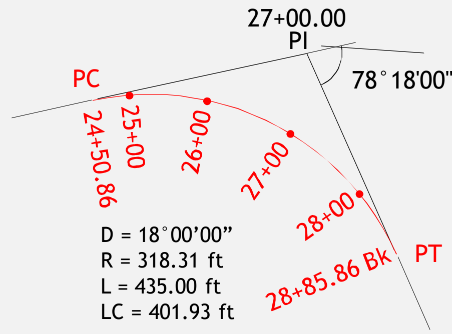
Example

defl rate = 18°00'00"/200 ft = 0.09°/ft

$l_i$ = $Sta_i$ = (24+50.86)

$\delta_i = l_i \times 0.09°/ft$

$r_i = 2(318.31)\sin(\delta_i)$




Horizontal Curves

E. Alignment Curves

4. Modified Deflection Angle Method

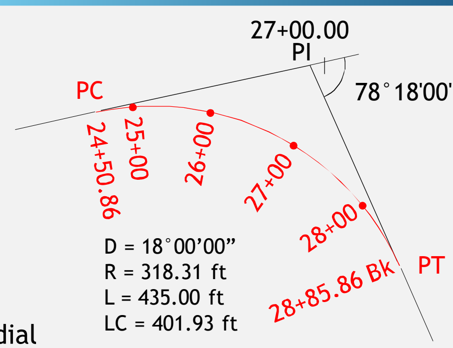
Example

defl rate = 18°00'00"/200 ft = 0.09°/ft

$l_i$ = $Sta_i$ = (24+50.86)

$\delta_i = l_i \times 0.09°/ft$

$r_i = 2(318.31)\sin(\delta_i)$




| Sta | arc, $l_i$ | Total def ang, $\delta_i$ | Radial chord, $r_i$ |
|---|---|---|---|
| PT Bk 28+85.86 | | | |
| 28+00 | | | |
| 27+00 | | | |
| 26+00 | | | |
| 25+00 | | | |
| PC 24+50.86 | **0.000** | 0°00'00" | 0.00 |

Horizontal Curves

# E. Alignment Curves

4. Modified Deflection Angle Method

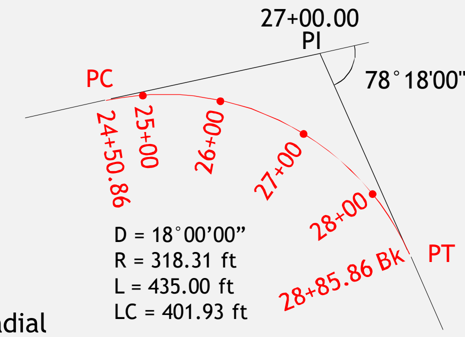
Example

defl rate = 18°00'00"/200 ft = 0.09°/ft

$l_i$ = $Sta_i$ = (24+50.86)

$\delta_i = l_i \times 0.09°/ft$

$r_i = 2(318.31)\sin(\delta_i)$




| Sta | arc, $l_i$ | Total def ang, $\delta_i$ | Radial chord, $r_i$ |
|---|---|---|---|
| PT Bk 28+85.86 | | | |
| 28+00 | | | |
| 27+00 | | | |
| 26+00 | | | |
| 25+00 | 49.14 | 4°25'21" | 49.09 |
| PC 24+50.86 | **0.000** | 0°00'00" | 0.00 |

Horizontal Curves

# E. Alignment Curves

4. Modified Deflection Angle Method

Example

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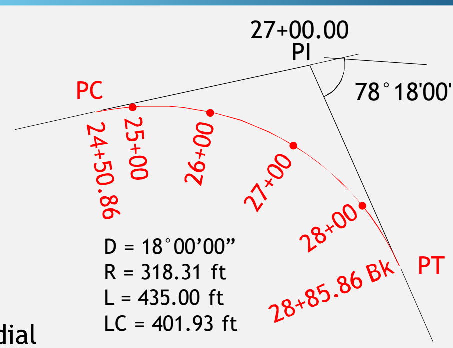


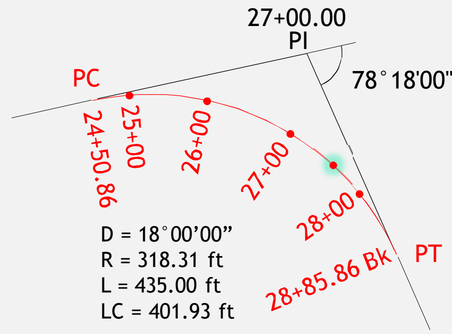

| Sta | arc, $l_i$ | Total def ang, $\delta_i$ | Radial chord, $r_i$ |
|---|---|---|---|
| PT Bk 28+85.86 | 435.00 =L | 39°09'00" =Δ/2 | 401.93 =LC |
| 28+00 | 349.14 | 31°25'21" | 331.90 |
| 27+00 | 249.14 | 22°25'21" | 242.83 |
| 26+00 | 149.14 | 13°25'21" | 147.78 |
| 25+00 | 49.14 | 4°25'21" | 49.09 |
| PC 24+50.86 | **0.000** | 0°00'00" | 0.00 |

Horizontal Curves

# E. Alignment Curves

4. Modified Deflection Angle Method

Example




Another advantage: can compute & stake *any* curve point.

Sta 27+60:

$l = (27+60)-(24+50.86) = 309.14$ ft

$\delta = 309.14 \text{ ft} \times 0.09°/\text{ft} = 27°49'21''$

$r = 2(318.31)\sin(27°49'21'') = 297.13$ ft

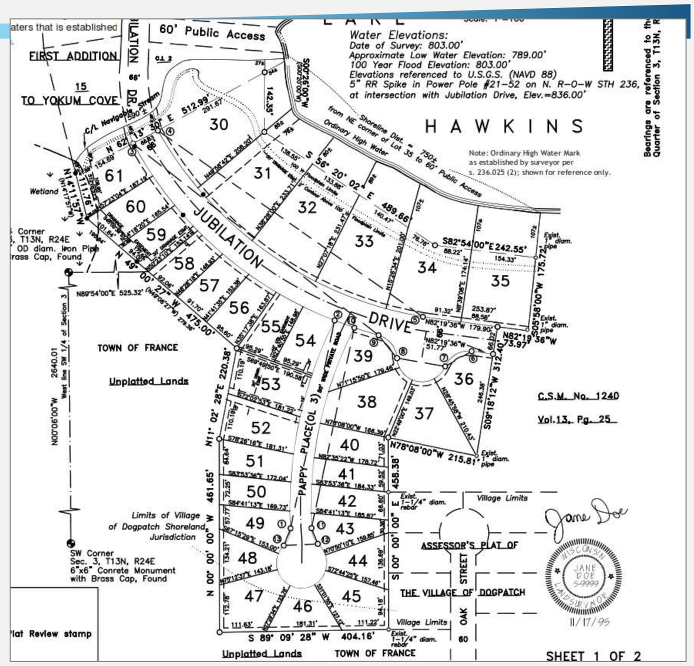
Horizontal Curves

# F. Curves and Traverses

a. Typical Uses

Tangent & non-tangent applications

Boundaries are mixture of straight and curved lines.



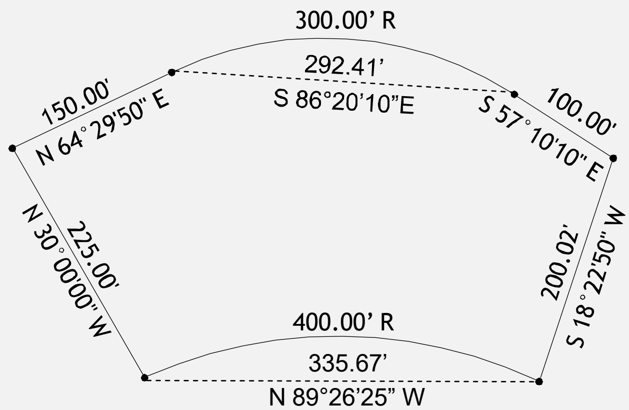

Horizontal Curves

## F. Curves and Traverses

b. Parcels

Curvilinear parcel boundaries.

Curves may or may not be tangent




Horizontal Curves

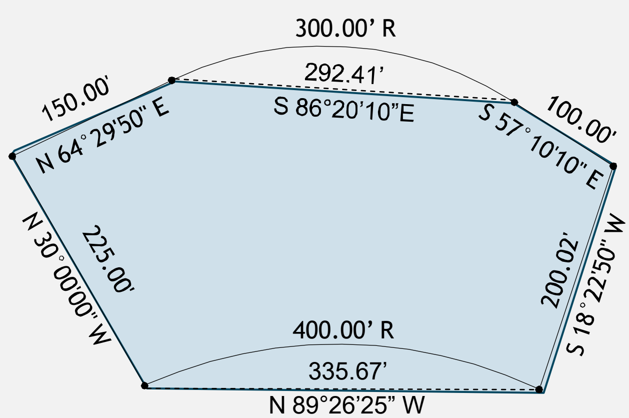
## F. Curves and Traverses

b. Parcels

Curvilinear parcel boundaries.

To determine parcel area:

(1) Compute area by coordinates bounded by straight lines




Horizontal Curves

## F. Curves and Traverses

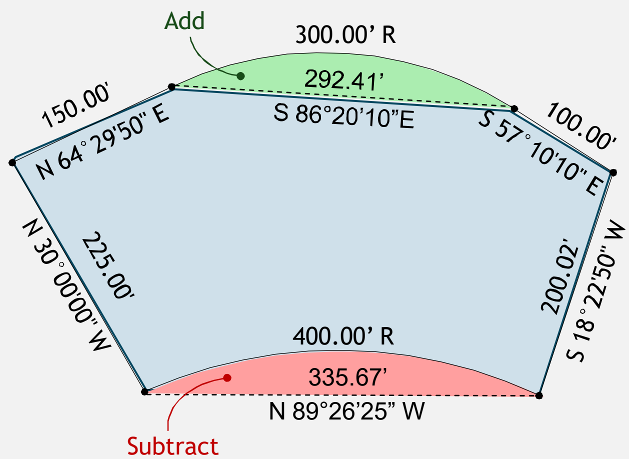
b. Parcels

Curvilinear parcel boundaries.

To determine parcel area:

(1) Compute area by coordinates bounded by straight lines

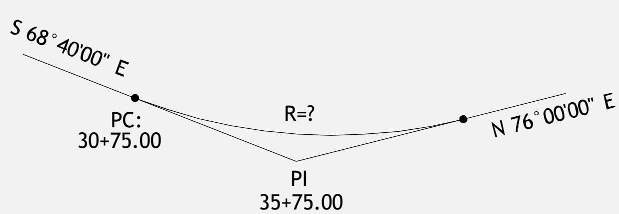
(2) Add segment area 1, subtract segment area 2




Horizontal Curves

## G. Problems

1. What curve radius meets this design?




Horizontal Curves

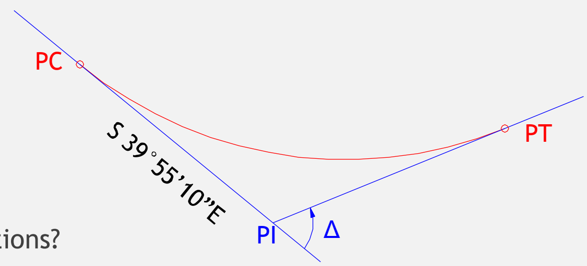
## G. Problems

2. Given:

PI Sta = 25+00.00

Δ = 65°00'00" L

R = 700.00 ft



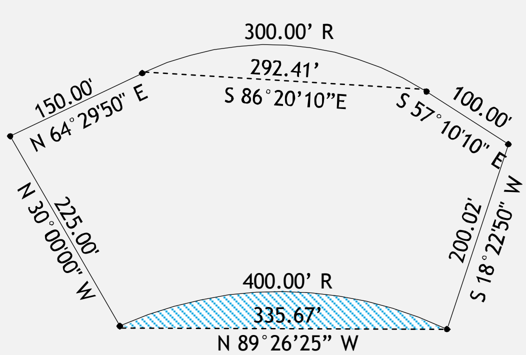

Part a. What are the PC & PT stations?

Part b. What is the chord distance between the first and last full stations on the curve?

Horizontal Curves

## G. Problems

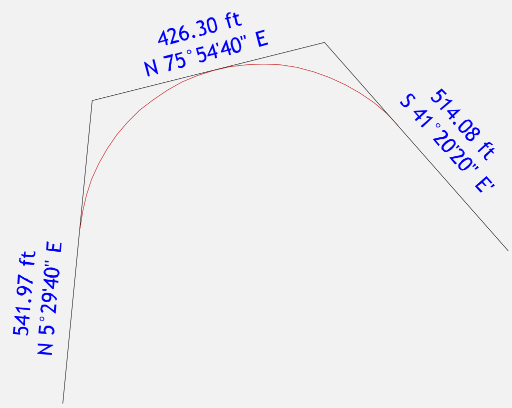
3. What is the area bounded by the southern curve and its chord?




Horizontal Curves

## G. Problems

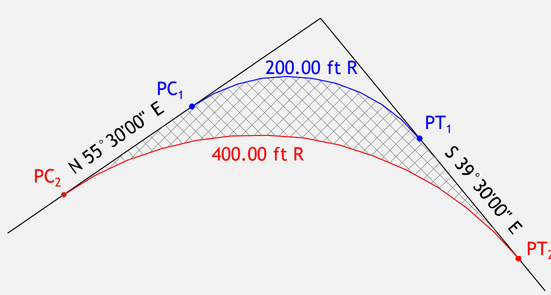
4. What is the radius of the curve which is tangent to all three lines?




Horizontal Curves

## G. Problems

5. What is the area between the two tangent circular arcs and tangent lines?




Horizontal Curves

# Questions?